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Seminar
Finite Groups And Their Representations

Topic: We repeat and deepen the theory of finite groups and learn basic and advanced topics in the representation theory of finite groups. This will be accompanied by working hands-on with several examples.

Time and Date: Summersemester 2014, Monday, 16:15-17:45, Geom. 430.

Prerequisites: The seminar aims at Master students, but is also open for Bachelor students, who have heard Algebra I. The parallel visit of Prof. Schweigert's lecture Algebra II will be beneficial, but not necessary. Students may also hold their talk in German language.

Organisational: The criterion for aquiring credits is giving a talk, handing in a manuscript and active participation in the exercise sessions. Please see me early about your talk and hand in a manuscript of your talk latest one week in advance. Our main literature will be "J. L. Alperin, R. B. Bell: Groups and representations."

Talks:

1. Simon Lentner: Repetition of basic definitions and examples. [AB] Chp. 1-3
   Subgroups, homomorphisms, normal subgroups, centralizer, commutator, conjugacy class, automorphisms.
   Exercises: abelian groups, $\mathbb{D}_4$, $\mathbb{S}_n$, $GL_n(\mathbb{F}_q)$.
2. NN: Permutation Action. [AB] Chp. 3 and 7
   Permutation action, Orbit-Stabilizer, Primitivity, Transitivity, Sylow theorem.
   Exercises: $\mathbb{D}_4$ on 4 points, $\mathbb{S}_5$ on 6 points, $GL_n(\mathbb{F}_q)$ on the projective points. Group order pq.
3. (NN:) The simple group(s) of order 60. [AB] Chp. 7.
4. (NN:) Construction of the smallest sporadic group, the Mathieu group M_{12}. [B] Chp. 1.2
5. NN: p-groups [AB] Chp. 8.
   Central series, Sylowgroups normal, Burnside Basis theorem [H].
   Exercises: Groups of order p^3. The q-Sylowgroup of $GL_n(\mathbb{F}_q)$.
6. NN: Solvable groups. [AB] Chp. 10,11.
   Central series, Jordan Hölder theorem, Frattini argument, Hall-Sylow-Systems [H].
   Exercises: $\mathbb{S}_3$, $\mathbb{S}_4$.

7. NN: Representation Theory. [AB] Chp. 12+13.
   Definitions, Characters, Sum Product, Maschke
   Exercises: $\mathbb{D}_4, \mathbb{S}_3$ representations and their products. Permutation representations.

8. NN: Character theory Theory. [AB] Chp. 14+15.
   Basic Operations, Orthogonality, Centralizers.
   Exercises: $\mathbb{D}_4, \mathbb{S}_3, \mathbb{A}_4, \mathbb{S}_4$ characters, products. Steinberg Rep. of $GL_n(\mathbb{F}_q)$ [C]

9. NN: Induction and Restriction. [AB] Chp. 16.
   Frobenius Reciprocity, Clifford restriction to normal subgroups [H]
   Exercises: $\mathbb{D}_4, \mathbb{S}_3$. Character table of $\mathbb{S}_5$ or $\mathbb{A}_5$, Principal series of $SL_2(\mathbb{F}_q)$.

10. NN: Representation of Coxeter groups. [C]
    Coxeter groups, McDonalds Induction.
    Exercise: $\mathbb{D}_4, \mathbb{D}_6, \mathbb{S}_3, \mathbb{S}_n$. To $Aut(S_6)$ via $\mathbb{S}_6 \cong Sp_4(\mathbb{F}_2)$. The Coxeter group $E_6 \cong PSU_4(2)$.

11. Simon Lentner: Outlook - Representation of finite Lie groups [C]
    Exercises: $SL_2(\mathbb{F}_2), SL_2(\mathbb{F}_3), SL_2(\mathbb{F}_{2^2}), SL_2(\mathbb{F}_3), SO_5(\mathbb{F}_2)$.

Literature:

- [AB] J. L. Alperin, R. B. Bell: Groups and representations, Graduate Texts in Mathematics 162 (1995), Springer.
- [B] J. Booher: Connections between the Mathieu Groups and Modular Forms, Bachelor thesis, Harvard 2010.
- [H] B. Huppert: Endliche Gruppen I (1984), Springer.
- [C] R. W. Carter: Representations of reductive groups, Cambridge University Press.